MIE-MPI: Tutorial 8

created: November 20, 2019, 12:09

8.1 Numerical mathematics

Machine numbers

If not stated otherwise, we consider **single precision** and mathematical rounding.

Exercise 8.1. Are the following numbers machine numbers?

a) 10^{113}

b) $1 + 2^{-32}$

c) $\frac{1}{5}$

d) $\frac{3}{10}$

e) $\frac{3}{256}$

f) $2^{-10} - 16$

Exercise 8.2. What are the (normalized) machine numbers which are the closest neighbours of the normalized machine number 2^t.

Exercise 8.3. Consider a decimal machine in which 2 decimal digits are allocated to the significand (and we do not care about the exponent). Sum the following numbers $0,25$, $0,0034$, $0,00051$ a $0,061$ in the following order:

a) from the least to the greatest,

b) from the greatest to the least.

Compare to the exact result.

Exercise 8.4. Let x and y be normalized machine numbers. Which following statements are true if we suppose that no underflow or overflow happens (and we stay within normalized numbers)?

1. $\mathrm{fl}(x + y) = \mathrm{fl}(y + x)$;
2. $\mathrm{fl}(xy) = \mathrm{fl}(yx)$;

3. $\mathrm{fl}(y - x) = -\mathrm{fl}(x - y)$;
4. $\mathrm{fl}(x + x) = \mathrm{fl}(2x)$;
5. $\mathrm{fl}(0.5x) = \mathrm{fl}(x/2)$;
6. $\mathrm{fl}((x + y) + c) = \mathrm{fl}(x + (y + c))$;
7. if $x \leq y$, then $x \leq \mathrm{fl}((x + y)/2) \leq y$.

If a statement is not true, find a counterexample. If it is true, give an argument. If your answer depends on something else, mention it.

Exercise 8.5. Let $p(x) = \sum_{i=0}^{n} a_i x^i \in \mathbb{R}[x]$. Assume that all numbers a_i lie in the range of normalized number and so does x. Give a bound on the error committed while calculating $p(x)$ assuming that no underflow or underflow happens during the calculation.