MIE-MPI: Tutorial 9

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9.1 Fuzzy set operations

Exercise 9.1. Consider fuzzy sets describing the speed of a car, ranging from 0 to 140 km/h. We have the following interpretations of the speed: *very slow*, *slow*, *medium*, *high*, *very high*. Their membership functions are continuous and partially linear functions given by the values in Table 9.1. Draw the graphs of these functions.

Table 9.1: Values of membership functions for car speeds. The function are linear between the specified points.

membership function	speed [km/h]														
	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140
very slow	1	1	0.5	0	0	0	0	0	0	0	0	0	0	0	
slow	0	0.5	1	1	1	0.5	0	0	0	0	0	0	0	0	0
medium	0	0	0	0	0	0.5	1	1	1	0.5	0	0	0	0	0
high	0	0	0	0	0	0	0	0	0.5	1	1	0.5	0	0	0
very high	0	0	0	0	0	0	0	0	0	0	0.5	1	1	1	1

Exercise 9.2. Write the definition and draw the graph of the membership function of the fuzzy set “high speed” $\cap$ “not very high speed” using the following t-norms:

a) Gödel,

b) Łukasiewicz,

c) product.

Exercise 9.3. Write the definition and draw the graph of the membership function of the fuzzy set “slow” $\cup$ “medium speed” using the following t-norms (and De Morgan’s law):

a) Gödel,

b) Łukasiewicz,

c) product.

Exercise 9.4. We want to construct a fuzzy controller of the car that tells us how much do we need to slow down (using the brakes) according to our current speed.

We have the following rules of the relation between speed and brake intensity:

R1 If the speed is very slow, then no braking.

R2 If the speed is slow, then no braking.

R3 If the speed is medium, then low braking.

R4 If the speed is high, then medium braking.

R5 If the speed is very high, then high braking.

The fuzzy sets describing the brake intensity are subsets of $[0, 100]$, interpreted as brake intensity in per cents. Their membership function are given as follows.

$$\nu_{\text{no}}(i) = \begin{cases} 1 & \text{for } i \in [0, 10], \\ \frac{30-i}{20} & \text{for } i \in [10, 30], \\ 0 & \text{for } i \in [30, 100], \end{cases} \tag{9.1}$$

$$\nu_{\text{low}}(i) = \begin{cases} 0 & \text{for } i \in [0, 10], \\ \frac{i-10}{20} & \text{for } i \in [10, 30], \\ \frac{50-i}{20} & \text{for } i \in [30, 50], \\ 0 & \text{for } i \in [50, 100], \end{cases} \tag{9.2}$$

$$\nu_{\text{medium}}(i) = \begin{cases} 0 & \text{for } i \in [0, 30], \\ \frac{i-30}{20} & \text{for } i \in [30, 50], \\ \frac{70-i}{20} & \text{for } i \in [50, 70], \\ 0 & \text{for } i \in [70, 100], \end{cases} \tag{9.3}$$

$$\nu_{\text{high}}(i) = \begin{cases} 0 & \text{for } i \in [0, 50], \\ \frac{i-50}{20} & \text{for } i \in [50, 70], \\ 1 & \text{for } i \in [70, 100]. \end{cases} \tag{9.4}$$

Determine the control sets when the speed is 50 km/h using the standard fuzzy controller with Mamdani implication (using Gödel t-norm). Calculate the defuzzyfied output at this speed (using the centre of gravity method).