

MIE-MPI: Tutorial 9

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9.1 Numerical mathematics

Machine numbers

If not stated otherwise, we consider **single precision** and mathematical rounding.

Exercise 9.1. Are the following numbers machine numbers?

- a) 10^{113}
- b) $1 + 2^{-32}$
- c) $\frac{1}{5}$
- d) $\frac{3}{10}$
- e) $\frac{3}{256}$
- f) $2^{-20} - 16$

Exercise 9.2. What are the (normalized) machine numbers which are the closest neighbours of the normalized machine number 2^t .

Exercise 9.3. Consider a decimal machine in which 2 decimal digits are allocated to the significand (and we do not care about the exponent). Sum the following numbers 0,25, 0,0034, 0,00051 a 0,061 in the following order:

- a) from the least to the greatest,
- b) from the greatest to the least.

Compare to the exact result.

Exercise 9.4. Find the absolute and the relative errors of the following pair (a, α) , where α is an approximate value of a :

- $(0.100, 0.110)$,
- $(0.100, 0.101)$,
- $(0, 500, 0.510)$,

- $(0.500, 0.501)$.

Exercise 9.5. Let x and y be normalized machine numbers. Which following statements are true if we suppose that no underflow or overflow happens (and we stay within normalized numbers)?

1. $\text{fl}(x + y) = \text{fl}(x) + \text{fl}(y)$;
2. $\text{fl}(x + y) = \text{fl}(y + x)$;
3. $\text{fl}((x + y) + c) = \text{fl}(x + (y + c))$;

If a statement is not true, find a counterexample. If it is true, give an argument. If your answer depends on something else, mention it.