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|----------------|------|----|----|-------------------|----------|
| MIE-MPI – EXAM |      |    |    | DECEMBER 17, 2021 |          |
| Name           | Q1–6 | Q7 | Q8 | Q9                | $\Sigma$ |
|                |      |    |    |                   |          |

|                                       |    |    |    |    |    |
|---------------------------------------|----|----|----|----|----|
| Multiple choice question answer table |    |    |    |    |    |
| Q1                                    | Q2 | Q3 | Q4 | Q5 | Q6 |
|                                       |    |    |    |    |    |

**Instructions:** Questions 1 to 6 have possible answers labelled A–E. There is always exactly one correct answer. Please, use the table above to mark your answer. If you make a mistake, correct your answer in the table (in a readable manner).

Other questions serve as a preparation for the oral part of the exam (nevertheless, your written preparation should be understandable). Don't forget to sign this sheet and all the sheets that you will hand in.

*You can use only paper, pen and **your** brain! Good luck!*

**Question 1** (5 points). How many non-negative integers strictly less than 20 can be equal to the order of some field?

- (A) 7.
- (B) 8.
- (C) 12.
- (D) 19.
- (E) 20.

**Question 2** (5 points). What is the value of the second mixed derivative  $\frac{\partial^2 f}{\partial x \partial y}(x, y)$  of the function  $f(x, y) = x^2 - xy + y^3$  at the point  $(-2, 2)$ ?

- (A) 0.
- (B) 2.
- (C) -12.
- (D) -1.
- (E) None of the above values.

**Question 3** (5 points). Which of the following polynomials is reducible (not irreducible) over  $\mathbb{Z}_3$ ?

- (A)  $x + 1$ .
  - (B)  $2x^2 + 2x + 1$ .
  - (C)  $x^3 + x^2 + x + 1$ .
  - (D)  $x^3 + x^2 + 2x + 1$ .
  - (E) None of the above polynomials.
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**Question 4** (5 points). Let us consider the permutation  $f = (279135684) \in S_9$ . The permutation  $f^{42}$  is

- (A) (6 5 4 2 9 3 5 8 1)
  - (B) (6 9 2 1 5 7 3 8 4)
  - (C) (8 2 9 4 7 6 1 5 3)
  - (D) (7 6 4 2 9 3 5 8 1).
  - (E) None of the above permutations.
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**Question 5** (5 points). Let us consider as domain  $D$  the triangle with vertices the points  $(0, 0)$ ,  $(2, 0)$  and  $(2, 1)$ . Select the value of the double integral

$$\iint_D 2x - y \, dx dy.$$

- (A) 10
  - (B)  $\frac{1}{8}$
  - (C)  $\frac{7}{3}$
  - (D) 0
  - (E) None of the above values.
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**Question 6** (5 points). Let  $A$  and  $B$  be two fuzzy sets (over a universe  $U$ ) having membership functions  $\mu_A$  and  $\mu_B$  respectively. Using the Łukasiewicz t-norm for intersection, give the formula of the membership function of  $A \cup B$ .

- (A)  $\mu_{A \cup B}(x) = 1 - \max\{0, 1 - \mu_A(x) - \mu_B(x)\}$
- (B)  $\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$
- (C)  $\mu_{A \cup B}(x) = 1 - \mu_A(x)\mu_B(x)$
- (D)  $\mu_{A \cup B}(x) = \mu_A(x) - \mu_B(x)$

(E) None of the above options is true.

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\*\*\* ORAL PART PREPARATION \*\*\*

**Question 7.** (10 points)

1. Write down the definitions of group and of subgroup.
  2. What is a cyclic group? Give an example of a group that is not cyclic.
  3. Can two groups have the same number of elements but different Cayley tables?
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**Question 8.** (10 points) Describe the single precision floating point number representation system.

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**Question 9.** (10 points) Let  $f, g : \mathbb{R}^2 \rightarrow \mathbb{R}$  be two functions and  $(x, y) \in \mathbb{R}^2$ . List sufficient conditions for  $(x, y)$  to be

- (a) a point of local strict minimum;
- (b) a point of local strict minimum subject to  $g(x, y)$ .