MIE-MPI – EXAM					January 19, 2022
Name	Q1–6	Q7	Q8	Q9	Σ

Multiple choice question answer table					
Q1	Q2	Q3	Q4	Q5	Q6

Instructions: Questions 1 to 6 have possible answers labelled A–E. There is always exactly one correct answer. Please, use the table above to mark your answer. If you make a mistake, correct your answer in the table (in a readable manner).

Other questions serve as a preparation for the oral part of the exam (nevertheless, your written preparation should be understandable). Don't forget to sign this sheet and all the sheets that you will hand in.

*You can use only paper, pen and **your** brain! Good luck!*

Question 1 (5 points)**.** What is the value of the second derivative $\frac{\partial^2 f}{\partial y^2}(x,y)$ of the function $f(x,y) = 3x - x^2y^2 + \ln y$ at the point $(1,-1)$?

(A) -1.

(B) 0.

(C) 1.

(D) 2.

(E) None of the above values.

Question 2 (5 points)**.** Select the correct statement.

(A) There exist group of order 4 that are not cyclic.

(B) $P(x) \in K[x]$ is irreducible over a field K if and only if it cannot be decomposed into a product of two elements of $K[x]$.

(C) Every group of order strictly less than 6 is cyclic.

(D) The group $\mathbb{Z}_{33}^{\times}$ is isomorphic to the group $\mathbb{Z}_{33}^{+}$.

(E) No other option is true.

Question 3 (5 points)**.** In the field $GF(3^2)$, where the multiplication is defined as multiplication modulo the irreducible polynomial x^2+1, find the inverse of 11.

(A) 101.

(B) 12.

(C) 21.

(D) 03.

(E) None of the above polynomials.

Question 4 (5 points)**.** Let us consider the permutation $f = (4\,3\,2\,6\,7\,8\,9\,1\,5) \in S_9$. The permutation f^{26} is

(A) $(4\,3\,2\,6\,7\,8\,9\,1\,5)$

(B) $(6\,2\,3\,8\,9\,1\,5\,4\,7)$

(C) $(8\,2\,3\,1\,5\,4\,7\,6\,9)$

(D) $(2\,5\,7\,3\,1\,9\,4\,6\,8)$.

(E) None of the above permutations.

Question 5 (5 points)**.** Let us consider as domain D the triangle with vertices the points $(0, 0)$, $(2, 0)$ and $(1, 1)$. Select the value of the double integral

$$\iint_D y^2 + x \,\mathrm{d}x\mathrm{d}y.$$

(A) -10

(B) $\frac{1}{4}$

(C) $\frac{5}{3}$

(D) 0

(E) None of the above values.

Question 6 (5 points)**.** Let A and B be two fuzzy sets (over a universe U) having membership functions μ_A and μ_B respectively. Using the product t-norm for intersection, give the formula of the membership function of $A \cup B^C$.

(A) $\mu_{A\cup B^C}(x) = 1 - \min\{0, 1 - \mu_A(x) - \mu_B(x)\}$

(B) $\mu_{A\cup B^C}(x) = \mu_A(x) - \mu_A(x)\mu_B(x)$

(C) $\mu_{A\cup B^C}(x) = \mu_A(x)\mu_B(x) - \mu_B(x) + 1$

(D) $\mu_{A\cup B^C}(x) = \mu_A(x) - \mu_B(x)$

(E) None of the above options is true.

*** ORAL PART PREPARATION ***

Question 7. (10 points)

(a) Write down the definition of group, cyclic group, and generator.

(b) Give an example of an infinite group which is not cyclic.

(c) How many generators can a group have? Relate this number to the order of the group (if the group is finite).

Question 8. (10 points) Describe the single precision floating point number representation system.

Question 9. (10 points) Let $f : \mathbb{R}^2 \to \mathbb{R}$. Let $(x, y) \in \mathbb{R}^2$ such that $\nabla f(x, y)$ exists and is zero. List sufficient conditions for (x, y) to be

(a) a point of local non-strict minimum;

(b) a point of local strict minimum;

(c) a saddle point.