MIE-MPI – EXAM					February 2, 2022
Name	Q1–6	Q7	Q8	Q9	Σ

Multiple choice question answer table					
Q1	Q2	Q3	Q4	Q5	Q6

Instructions: Questions 1 to 6 have possible answers labelled A–E. There is always exactly one correct answer. Please, use the table above to mark your answer. If you make a mistake, correct your answer in the table (in a readable manner).

Other questions serve as a preparation for the oral part of the exam (nevertheless, your written preparation should be understandable). Don't forget to sign this sheet and all the sheets that you will hand in.

*You can use only paper, pen and **your** brain! Good luck!*

Question 1 (5 points)**.** What is the value of the second mixed derivative $\frac{\partial^2 f}{\partial x \partial y}(x,y)$ of the function $f(x,y) = 4x - x^2 y + \ln y$ at the point $(-1,1)$?

(A) -1.

(B) 0.

(C) 1.

(D) 2.

(E) None of the above values.

Question 2 (5 points)**.** Let us consider as domain D the triangle with vertices the points $(0,0)$, $(1,0)$ and $(0,2)$. Select the value of the double integral

$$\iint_D x - y \,\mathrm{d}x\mathrm{d}y.$$

(A) $-\frac{1}{3}$

(B) -1

(C) 0

(D) $\frac{1}{4}$

(E) None of the above values.

Question 3 (5 points)**.** Select the correct statement.

(A) Every group of order 4 is cyclic.

(B) Infinite groups can only be cyclic.

(C) The group $\mathbb{Z}_3^\times$ is isomorphic to the group $\mathbb{Z}_2^+$.

(D) $P(x) \in K[x]$ is irreducible over a field K if and only if it cannot be decomposed into a product of two elements of $K[x]$.

(E) No other option is true.

Question 4 (5 points)**.** In the field $GF(2^2)$, where the multiplication is defined as multiplication modulo the irreducible polynomial $x^2 + x + 1$, find the inverse of 11.

(A) 101.

(B) 12.

(C) 10.

(D) 11.

(E) None of the above polynomials.

Question 5 (5 points)**.** Let us consider the permutation $f = (5\,2\,1\,6\,3\,4\,7) \in S_7$. The permutation f^9 is

(A) $(3\,2\,5\,4\,1\,6\,7)$

(B) $(2\,5\,7\,3\,1\,9\,4\,6\,8)$.

(C) $(1\,2\,3\,6\,5\,4\,7)$

(D) $(5\,2\,1\,6\,3\,4\,7)$

(E) None of the above permutations.

Question 6 (5 points)**.** Let A and B be two fuzzy sets (over a universe U) having membership functions μ_A and μ_B respectively. Using the Gödel t-norm for intersection, give the formula of the membership function of $A \cup B^C$.

(A) $\mu_{A\cup B^C}(x) = 1 - \min\{1 - \mu_A(x), \mu_B(x)\}$

(B) $\mu_{A\cup B^C}(x) = \min\{1 - \mu_A(x), 1 - \mu_A(x)\mu_B(x)\}$

(C) $\mu_{A\cup B^C}(x) = \mu_A(x)\mu_B(x) - \mu_B(x) + 1$

(D) $\mu_{A\cup B^C}(x) = \min\{\mu_A(x), \mu_B(x)\}$

(E) None of the above options is true.

*** ORAL PART PREPARATION ***

Question 7. (10 points) Let $f : \mathbb{R}^3 \to \mathbb{R}$. Let $(x, y, z) \in \mathbb{R}^2$ such that $\nabla f(x, y, z)$ exists and is zero. List sufficient conditions for (x, y, z) to be

(a) a point of local maximum;

(b) a point of local minimum;

(c) a saddle point.

Question 8. (10 points)

(a) Write down the definition of ring and of field.

(b) Give an example of a field that is not a ring.

(c) What is the possible order of a field?

Question 9. (10 points) Describe the single precision floating point number representation system.