NIE-MPI, Mathematics for Informatics - Homework no. 1

Instructions:

- You should try to solve all the exercises. Even if you do not do all the exercises, you can get all the points.
- Presentation is taken into account; correct results themselves are not enough. The reasoning on how the result was found should be clearly visible.
- Comment your calculations in a reasonable way: the reader should understand what you do and *why*. The solution should be "possible to read", not "needed to decrypt".
- Do not answer unasked questions. It is important to know what is needed to solve the problem and what is not needed.
- If you use a result from another source than the lectures and tutorials, cite your source properly (do not forget to cite used software if applicable).
- The homework should be given by hand or sent by email at `dolcefra@fit.cvut.cz` before the beginning of the lecture on Wednesday October 27th, 2021.

Exercise 1. Find all minima, maxima and saddle points of the following functions:

(a) $f(x, y) = x^2 + xy + 2y^2$,

(b) $f(x, y) = x^3 + xy + y^3$,

(c) $f(x, y) = \frac{1}{x} + 4xy$,

(d) $f(x, y) = x \ln x + y \ln y + \sqrt{3}$,

(e) $f(x, y, z) = x^2 + 4xy^4 + 4x^2z + 2x^2y^2 - z^3$.

Exercise 2. Find minima, maxima and saddle points of the following functions:

(a) $f(x, y) = x^2 - y$ subject to $x^2 + \frac{1}{4}y^2 = 1$,

(b) $f(x, y, z) = xy + yz + y$ subject to $xyz = 1$,

Exercise 3. Calculate

$$\iint_D xy \, \mathrm{d}x \, \mathrm{d}y$$

where D is equal to:

(a) $[-2, 2] \times [0, 2]$;

(b) the triangle with vertices $(0, 1), (1, 3)$ and $(2, 0)$;

(c) the bounded subset of $(\mathbb{R}^+)^2$ which is delimited by the y-axis, the curve having equation $y = x^2$ and the line having equation $y = 3 - 2x$.