NIE-MPI: Tutorial 3

created: October 6, 2021, 16:43

3.1   Constrained optimization

Exercise 3.1. Find all the local maxima and minima of the function $f(x, y) = 3x - 4y + 3$ subject to

$$x^2 + y^2 = 4.$$

Exercise 3.2. Find all the local maxima and minima of the function $f(x, y) = xy$ subject to

$$x + y = 1.$$

Exercise 3.3. Find all the local maxima and minima of the function $f(x, y) = x^2 + y^2$ subject to

$$\frac{x}{a} + \frac{y}{b} = 1,$$

where a and b are non-zero real numbers.

Exercise 3.4. Find all the local maxima and minima of the function $f(x, y) = 2x^2 - 2y^2$ subject to

$$y + e^{-x^2} = 1.$$

3.2   2-variate function integration

Exercise 3.5. Let $f(x, y) = e^{2x+y}$ and $D = [0, 1] \times [0, 3]$. Evaluate

$$\iint_D f(x, y) \, dx \, dy.$$

Exercise 3.6. Let $f(x, y) = \sin(x + y)$ and $D = [0, \pi] \times [0, 2\pi]$. Evaluate

$$\iint_D f(x, y) \, dx \, dy.$$

Exercise 3.7. Find the volume of the solid object delimited by the graph of

$$f(x, y) = x^2 + y^2$$

and by the planes $x = 0, x = 3, y = -1, y = 1$ and $z = 0$.

Exercise 3.8. Let $f(x, y, z) = x + 2y + 3z$ and $D = [0, 1] \times [-\frac{1}{2}, 0] \times [0, \frac{1}{3}]$. Evaluate

$$\iiint_D f(x, y, z) \, dx \, dy \, dz.$$

Exercise 3.9. Let $f(x, y, z) = e^{x+y+z}$ and $D = [0, 1] \times [0, 1] \times [0, 1]$. Evaluate

$$\iiint_D f(x, y, z) \, dx \, dy \, dz.$$

Integrals over non-rectangular domain

Exercise 3.10. Evaluate

$$\iint_D (x + y) \, dx \, dy \,,$$

where D is the domain delimited by the graph of $y = x^2$ for $x \in [0, \frac{1}{2}]$ and the x-axis.

Exercise 3.11. Evaluate

$$\iint_D (x + y)^2 \, dx \, dy$$

where D is the triangular surface with vertices $(0, 0), (0, 1)$ and $(2, 2)$.

Exercise 3.12. Evaluate

$$\int_0^1 \int_x^1 xy \, dy \, dx \,.$$

Exercise 3.13. Evaluate

$$\int_0^1 \int_{1-y}^1 (x + y^2) \, dx \, dy \,.$$

Exercise 3.14. Evaluate

$$\iint_D (x - y) \, dx \, dy \,,$$

where D is the triangular surface with vertices $(0, 0), (1, 0)$ and $(2, 1)$.