| NIE-MPI - EXAM | JANUARY 10, 2023 |  |  |  |  |
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| Name | Q1-6 | Q7 | Q8 | Q9 | $\boldsymbol{\Sigma}$ |
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| Multiple choice question answer table |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Q1 | Q2 | Q3 | Q4 | Q5 | Q6 |
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Instructions: Questions 1 to 6 have possible answers labelled A-E. There is always exactly one correct answer. Please, use the table above to mark your answer. If you make a mistake, correct your answer in the table (in a readable manner).

Other questions serve as a preparation for the oral part of the exam (nevertheless, your written preparation should be understandable). Don't forget to sign this sheet and all the sheets that you will hand in.

You can use only paper, pen and your brain! Good luck!

Question 1 (5 points). What is the value of the second derivative with respect to $x$ of the function $f(x, y)=\sqrt{x}-x^{2} y+\ln y$ at the point $(1,2)$ ?
(A) 3 .
(B) -2 .
(C) 0 .
(D) $-\frac{1}{2}$.
(E) None of the above values.

Question 2 ( 5 points). Let us consider as domain $D$ the finite region delimited by the graph $y=2 x-x^{2}$ and the $x$-axis. Select the value of the double integral

$$
\iint_{D} x-y \mathrm{~d} x \mathrm{~d} y
$$

(A) $\frac{7}{4}$
(B) $\frac{4}{5}$
(C) -4
(D) 0
(E) None of the above values.

Question 3 (5 points). How many generators has the group $\mathbb{Z}_{20}^{+}$?
(A) 7 .
(B) 19 .
(C) 8 .
(D) 20 .
(E) 10 .

Question 4 ( 5 points). Let $A$ and $B$ be two fuzzy sets (over a universe U ) having membership functions $\mu_{A}$ and $\mu_{B}$ respectively. Using the Łukasiewicz t-norm for intersection, give the formula of the membership function of $A \cup B$.
(A) $\mu_{A \cup B}(x)=-\left(\max \left\{-\mu_{A}(x)-\mu_{B}(x)+1,0\right\}-1\right)$
(B) $\mu_{A \cup B}(x)=\max \left\{\mu_{A}(x), \mu_{B}(x)\right\}+1$
(C) $\mu_{A \cup B}(x)=1-\mu_{A}(x) \mu_{B}(x)$
(D) $\mu_{A \cup B}(x)=\mu_{A}(x)-\mu_{B}(x)$
(E) None of the above options is true.

Question 5 (5 points). In the field $G F\left(5^{2}\right)$ with multiplication modulo $x^{2}+4 x+1$, find the inverse of 13 .
(A) 33
(B) 02
(C) 112
(D) 51
(E) None of the above option.

Question 6 (5 points). Let us consider the permutation $f=(4253176) \in S_{7}$. The permutation $f^{22}$ is
(A) $(3215467)$
(B) $(4253176)$
(C) $(3541672)$
(D) (5142763).
(E) None of the above permutations.

## *** ORAL PART PREPARATION ${ }^{* * *}$

Question 7. (10 points)

1. Write down the definition of monoid, semigroup and group.
2. Is it possible to construct a cyclic group of any order? Justify your answer.

Question 8. (10 points) Let $f, g: \mathbb{R}^{2} \rightarrow \mathbb{R}$. List sufficient conditions for a point $(x, y) \in \mathbb{R}^{2}$ to be
(a) a point of local strict minimum of $f$ subject to $g$;
(b) a point of local strict maximum of $f$ subject to $g$;
(c) a saddle point of $f$ subject to $g$.

Question 9. (10 points) Describe the single precision floating point number representation system.

