NIE-MPI, Mathematics for Informatics - Homework no. 3

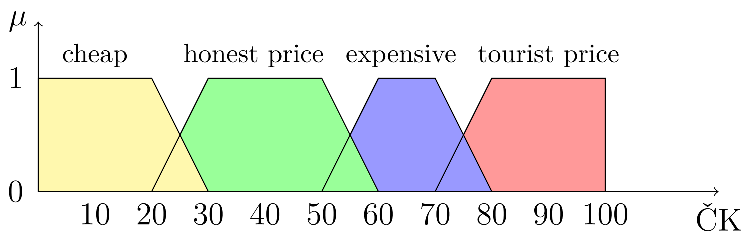
Instructions:

- You should try to solve all the exercises. Even if you do not do all the exercises, you can get all the points.
- Presentation is taken into account; correct results themselves are not enough. The reasoning on how the result was found should be clearly visible.
- Comment your calculations in a reasonable way: the reader should understand what you do and *why*. The solution should be "possible to read", not "needed to decrypt".
- Do not answer unasked questions. It is important to know what is needed to solve the problem and what is not needed.
- If you use a result from another source than the lectures and tutorials, cite your source properly (do not forget to cite used software if applicable).
- For Exercise 6, choose a programming language from the list { C, C++, Python, SageMath, MATLAB, Java, JavaScript, Pascal }. The source code must contain comments to understand what is happening in the program. Before sending the source code, check that the program actually compiles.
- The homework should be sent by email at `dolcefra@fit.cvut.cz` before Monday December 19th, 2022 (included).

Exercise 1. Prove that the following binary operation is a t-norm:

$$x \star y = \begin{cases} y & \text{if } x = 1, \\ x & \text{if } y = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Exercise 2. Let us consider the 4 fuzzy sets in the following picture describing beer's price in Prague: *cheap*, *honest price*, *expensive*, *tourist price*.



Write the definition and draw the graph of the membership function of the fuzzy sets "**Special occasion beer**" defined us "*expensive*" ∩ "*tourist price*" using the following t-norms:

a) Gödel,

b) product,

c) Łukasiewicz.

Exercise 3. Using the picture and the fuzzy sets of the previous exercice (and De Morgan's law) write the definition and draw the graph of the membership function of the fuzzy set "**Friday beer**" defined as "*cheap*" $\cup$ "*honest price*" using the following t-norms:

a) Gödel,

b) product,

c) Łukasiewicz.

Exercise 4. Say which of the following numbers are normalized machine numbers in single precision (`binary32`) or in double precision (`binary64`). When is the case, represent them (using the standard `IEEE-754`).

- 10^{3005};
- 10^{-3005};
- $9 + \frac{5}{8}$;
- 8^6;
- 8^{-6};
- $\frac{7}{5}$;
- $5 + \frac{1}{5}$;
- $5 + \frac{1}{8}$;
- 2^{-1280};
- 2^{-128}.

Exercise 5. Is it true that $\mathrm{fl}(x + y) = \mathrm{fl}(x) + \mathrm{fl}(y)$? Justify your answer (i.e., give an argument or find a counterexample).

Exercise 6. Write a program giving, using an iterative method, a solution of the system of linear equations $\mathbf{A}\mathbf{x} = \mathbf{b}$, where:

- $\mathbf{A}$ is the coefficient matrix of the system;
- $\mathbf{b}$ is the constant vector of the system;
- $\mathbf{x}$ is the desired vector solution.

To solve the system use the Jacobi method. Set the stop criterion of your program so that (when the method converges) the program returns an approximate solution $\mathbf{x}^*$ satisfying

$$\frac{\|\mathbf{A}\mathbf{x}^* - \mathbf{b}\|_2}{\|\mathbf{b}\|_2} < 10^{-6},$$

where $\|\cdot\|_2$ represents the Euclidean norm.

Use the following inputs for your program:

$$\mathbf{A} = \begin{pmatrix} \alpha & -1 & & & \\ -1 & \alpha & -1 & & \\ & -1 & \alpha & \ddots & \\ & & \ddots & \ddots & -1 \\ & & & -1 & \alpha \end{pmatrix} \in \mathbb{R}^{20,20}, \quad \mathbf{b} = \begin{pmatrix} \alpha - 1 \\ \alpha - 2 \\ \alpha - 2 \\ \vdots \\ \alpha - 2 \\ \alpha - 2 \\ \alpha - 1 \end{pmatrix} \in \mathbb{R}^{20},$$

where there are zeros in the empty spaces in the matrix $\mathbf{A}$. As initial vector use $\mathbf{x}_0 = \mathbf{0}$.

Perform the calculation for the following three values of the parameter α:

a) $\alpha = 5$,

b) $\alpha = 2$,

c) $\alpha = \frac{1}{2}$.

How many iteretations are required, for each of the given values, to obtain the desired accuracy? Is the method converging for all values? If not, explain why.