

# NIE-MPI: Tutorial 2

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## 2.1 Critical points

**Exercise 2.1.** Determine all the points where the function  $\sqrt{x^2 + y^2}$  does not have partial derivatives.

**Exercise 2.2.** Find all the critical points of the following functions.

(a)  $f(x, y) = x^2 + 2y^2$ ,

(b)  $f(x, y, z) = 3x^2 + xy + 3zy$ ,

(c)  $f(x, y) = e^{-x^2 - 7y^2 + 3}$ ,

(d)  $f(x, y) = x^2 + y^2 + 3xy + 10$ ,

(e)  $f(x, y) = \sin(x^2 + y^2)$ ,

(f)  $f(x, y) = x^2 + xy^3 - \ln(x)$ .

## 2.2 Hessian matrix, extremal values

**Exercise 2.3.** Find the Hessian matrix of the following functions.

(a)  $f(x, y) = x^2y^2$ ,

(b)  $f(x, y) = e^{-(x+y)}$ ,

(c)  $f(x, y, z) = x^2 + y^3 + z^4$ .

**Exercise 2.4.** For which values of  $a$  is the matrix

$$\begin{pmatrix} 4 & 1 \\ 1 & a \end{pmatrix}$$

positively (semi)definite, negatively (semi)definite, indefinite.

## 2.3 Minima and maxima

**Exercise 2.5.** Find all local minima and maxima and saddle points of the following functions.

(a)  $f(x, y) = x^2 + 3xy + y^2 + 16,$

(b)  $f(x, y) = 3x^2 - 5xy + 3y^2,$

(c)  $f(x, y) = (x^2 - y^2)e^{(-x^2 - y^2)/2},$

(d)  $f(x, y) = e^{1+x^2-y^2},$

(e)  $f(x_1, x_2, x_3) = x_1^2 + 3x_2^2 - 3x_1x_2 + 4x_2x_3 + 6x_3^2,$

(f)  $f(x_1, x_2, x_3) = x_1x_3 + x_1^2 - x_2 + x_2x_3 + x_2^2 + 3x_3^2.$