NIE-MPI – EXAM	January 25, 2024				
Name	Q1–6	Q7	Q8	Q9	$\Sigma$

Multiple choice question answer table							
Q1	Q2	Q3	Q4	Q5	Q6		

**Instructions:** Questions 1 to 6 have possible answers labelled A–E. There is always exactly one correct answer. Please, use the table above to mark your answer. If you make a mistake, correct your answer in the table (in a readable manner).

Other questions serve as a preparation for the oral part of the exam (nevertheless, your written preparation should be understandable). Don't forget to sign this sheet and all the sheets that you will hand in.

You can use only paper, pen and your brain! Good luck!

Question 1 (5 points). How many non-negative integers strictly less than 30 can be equal to the order of some field?

- (A) 6.
- (B) 10.
- (C) 16.
- (D) 19.
- (E) 30.

**Question 2** (5 points). What is the value of the second mixed derivative  $\frac{\partial^2 f}{\partial y \partial x}(x, y)$  of the function  $f(x, y) = x^3 - xy + y^2$  at the point (3, -3)?

- (A) -12.
- (B) -1.
- (C) 0.
- (D) 3.
- (E) None of the above values.

Question 3 (5 points). Which of the following polynomials is irreducible over  $\mathbb{Z}_3$ ?

(A)  $2x^2 + 1$ .

- (B)  $x^2 + x + 1$ .
- (C)  $x^3 + x^2 + 1$ .
- (D)  $x^3 + x^2 + 2x + 1$ .
- (E) None of the above polynomials.

Question 4 (5 points). Let us consider the permutation  $f = (427913568) \in S_9$ . The permutation  $f^{42}$  is

- (A) (654293581)
- (B) (925847136)
- (C) (829476153)
- (D) (764293581).
- (E) None of the above permutations.

**Question 5** (5 points). Let us consider as domain D the triangle with vertices the points (0,0), (2,0) and (2,1). Select the value of the double integral

$$\iint_D 2x + y \, dx dy.$$

- (A) 10
- (B)  $\frac{1}{8}$
- (C) 3
- (D) 0
- (E) None of the above values.

Question 6 (5 points). Let A and B be two fuzzy sets (over a universe U) having membership functions  $\mu_A$  and  $\mu_B$  respectively. Using the Łukasiewicz t-norm for intersection, give the formula of the membership function of  $A \cup B^c$ .

- (A)  $\mu_{A \cup B}(x) = 1 \max\{0, \mu_B(x) \mu_A(x)\}\$
- (B)  $\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$
- (C)  $\mu_{A \cup B}(x) = 1 \mu_A(x)\mu_B(x)$
- (D)  $\mu_{A \cup B}(x) = \mu_A(x) \mu_B(x)$
- (E) None of the above options is true.

## \*\*\* ORAL PART PREPARATION \*\*\*

**Question 7.** (10 points) Let  $f, g : \mathbb{R}^2 \to \mathbb{R}$  be two functions and  $(x, y) \in \mathbb{R}^2$ . List sufficient conditions for (x, y) to be

- (a) a point of local strict maximum;
- (b) a point of local strict maximum subject to g(x, y).

## Question 8. (10 points)

- 1. Write down the definitions of group and of subgroup.
- 2. What is a cyclic group? Give an example of a group that is not cyclic.
- 3. Can two groups have the same number of elements but different Cayley tables?

Question 9. (10 points) Describe the single precision floating point number representation system.