NIE-MPI, Mathematics for Informatics - Homework no. 1

Instructions:

- You should try to solve all the exercises. Even if you do not do all the exercises, you can get all the points.
- Presentation is taken into account; correct results themselves are not enough. The reasoning on how the result was found should be clearly visible.
- Comment your calculations in a reasonable way: the reader should understand what you do and *why*. The solution should be "possible to read", not "needed to decrypt".
- Do not answer unasked questions. It is important to know what is needed to solve the problem and what is not needed.
- If you use a result from another source than the lectures and tutorials, cite your source properly (do not forget to cite used software if applicable).
- The homework should be given by hand or sent by email at `dolcefra@fit.cvut.cz` before the beginning of the tutorial on Thursday October 19th, 2023.

Exercise 1. Find all minima, maxima and saddle points of the following functions:

(a) $f(x, y) = 3x^2 - xy + y^2$,

(b) $f(x, y) = \frac{1}{5x} - 3xy$,

(c) $f(x, y) = (x + 2y - 5)^2$,

(d) $f(x, y) = -x \ln x + y^2 \ln x + \pi$,

(e) $f(x, y, z) = x^4 + y^3 - 16 \ln x - 9 \ln y$.

Exercise 2. Find minima, maxima and saddle points of the following functions:

(a) $f(x, y) = 2x^2 - y^2 + 6xy$ subject to $x + y = 2$,

(b) $f(x, y, z) = xz + yz + z$ subject to $xyz = 1$.

Exercise 3. Calculate

$$\iint_D (xy^3 + x^2y^2 - 1)\,\mathrm{d}x\,\mathrm{d}y$$

where D is equal to:

(a) $[0, 1] \times [0, 2]$;

(b) the triangle with vertices $(1, 0), (1, 1)$ and $(3, 0)$;

(c) the bounded subset of $(\mathbb{R}^+)^2$ which is delimited by the y-axis, the curve having equation $y = x^2$ and the line having equation $y = 3 - 2x$.