

# NIE-MPI: Tutorial 6

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## 6.1 Homomorphisms and isomorphisms

**Exercise 6.1.** Which of the following mappings is a homomorphism and which is an isomorphism from the given group to the given group?

- (a)  $f(n) = 3n + 2$ , from  $(\mathbb{Z}, +)$  to  $(\mathbb{R}, +)$ ;
- (b)  $f(x) = 2^x$ , from  $(\mathbb{R}, +)$  to  $(\mathbb{R}^+, \cdot)$ ;
- (c)  $f(A) = A_{1,1}$ , from the group of  $n \times n$  matrices with the matrix addition (element-wise), denoted  $(M, +)$ , to  $(\mathbb{R}, +)$ ;
- (d)  $f(A) = A_{1,1}$ , from the group of  $n \times n$  regular matrices with the matrix multiplication, denoted  $(M_{\text{reg}}, \cdot)$ , to  $(\mathbb{R} \setminus \{0\}, \cdot)$ .

**Exercise 6.2.** Find some homomorphism from  $(M_{\text{reg}}, \cdot)$  to  $(\mathbb{R} \setminus \{0\}, \cdot)$ .

**Exercise 6.3.** Is  $\mathbb{Z}_7^\times$  isomorphic with  $\mathbb{Z}_6^+$ ? If yes, find an isomorphism.

**Exercise 6.4.** How can we find an isomorphism between the groups  $\mathbb{Z}_p^\times$  and  $\mathbb{Z}_{p-1}^+$  when  $p$  is a prime number? How many different isomorphisms exist?

## 6.2 Permutations

**Exercise 6.5.** Let us consider the two following permutations in  $S_5$ :

$$f = (24513) \quad \text{and} \quad g = (54321).$$

- (a) Find  $g \circ f$ .
- (b) What is the order of the subgroup  $\langle f \rangle$  of  $S_5$ ? And the order of  $\langle g \rangle$ ?
- (c) Find  $f^{37} \circ g^{42}$ .