BIE-PST Cheat Sheet

FIT – CTU in Prague
Winter Semester 2023/2024

Probability	$\mathrm{P}(A) = \frac{\lvert A\rvert}{\lvert\Omega\rvert} = \frac{\#\text{ of favorable outcomes}}{\#\text{ of all outcomes}}$ or $\frac{\text{size of favorable outcomes}}{\text{size of all outcomes}}$
Conditional probability	$\mathrm{P}(A\|B) = \frac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}$
Multiplicative law	$\mathrm{P}(A_1 \cap \cdots \cap A_n) = \mathrm{P}(A_1)\,\mathrm{P}(A_2\|A_1)\cdots\mathrm{P}(A_n\|A_1\cap\cdots\cap A_{n-1})$
Law of total probability	$\mathrm{P}(A) = \sum_j \mathrm{P}(A\|B_j)\mathrm{P}(B_j)$
Bayes' theorem	$\mathrm{P}(B_i\|A) = \frac{\mathrm{P}(A\|B_i)\,\mathrm{P}(B_i)}{\sum_j \mathrm{P}(A\|B_j)\mathrm{P}(B_j)}$
Independence	$\mathrm{P}(A\cap B) = \mathrm{P}(A)\,\mathrm{P}(B)$

Distribution function	$F_X(x) = \mathrm{P}(X \le x)$

Discrete random variable X:

Probabilities of values	$\mathrm{P}(X = x)$
Expectation	$\mathrm{E}\,X = \sum\limits_k x_k\,\mathrm{P}(X = x_k)$
Expectation of a function	$\mathrm{E}\,g(X) = \sum\limits_k g(x_k)\,\mathrm{P}(X = x_k)$

Continuous random variable X:

Density	$f_X(x)$
Expectation	$\mathrm{E}\,X = \int\limits_{-\infty}^{+\infty} x f_X(x)dx$
Expectation of a function	$\mathrm{E}\,g(X) = \int\limits_{-\infty}^{+\infty} g(x) f_X(x)dx$
Variance X	$\mathrm{var}(X) = \mathrm{E}(X - \mathrm{E}\,X)^2 = \mathrm{E}\,X^2 - (\mathrm{E}\,X)^2$

Distribution	probabilities/density	expectation	variance
Bernoulli	$\mathrm{P}(X=1) = p, \mathrm{P}(X=0) = 1-p$	p	$p(1-p)$
Binomial	$\mathrm{P}(X=k) = \binom{n}{k} p^k (1-p)^{n-k}, k = 0, ..., n$	np	$np(1-p)$
Geometric	$\mathrm{P}(X=k) = (1-p)^{k-1}p, k \in \mathbb{N}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Poisson	$\mathrm{P}(X=k) = \frac{\lambda^k}{k!}e^{-\lambda}, k \in \mathbb{N}_0$	λ	λ
Uniform	$f(x) = \frac{1}{b-a}, x \in [a,b]$, elsewhere 0	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential	$f(x) = \lambda e^{-\lambda x}, x \ge 0$, elsewhere 0	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Normal	$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, x \in \mathbb{R}$	μ	σ^2

Joint distribution function $F_{X,Y}(x,y) = \mathrm{P}(X \le x \cap Y \le y)$	
Discrete X and Y	*Continuous X and Y*
Joint probabilities $\mathrm{P}(X = x \cap Y = y)$	Joint density $f_{X,Y}(x,y)$
Marginal distribution X	
$\mathrm{P}(X = x) = \sum_{y} \mathrm{P}(X = x \cap Y = y)$	$f_X(x) = \int_{-\infty}^{+\infty} f_{X,Y}(x,y)dy$
Independence of X and Y	
$\mathrm{P}(X = x \cap Y = y) = \mathrm{P}(X = x)\,\mathrm{P}(Y = y)$	$f_{X,Y}(x,y) = f_X(x) f_Y(y)$
Expectation of $\mathrm{E}\, g(X,Y)$	
$\sum_{x,y} g(x,y)\,\mathrm{P}(X = x \cap Y = y)$	$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(x,y) f_{X,Y}(x,y)dxdy$
Conditional distribution of X given $Y = y$:	
$\mathrm{P}\left(X = x \mid Y = y\right) = \frac{\mathrm{P}(X = x \cap Y = y)}{\mathrm{P}(Y = y)}$	$f_{X\mid Y}(x\mid y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$
Conditional expectation of X given $Y = y$:	
$\mathrm{E}(X\mid Y = y) = \sum_{x} x\,\mathrm{P}\left(X = x \mid Y = y\right)$	$\mathrm{E}(X\mid Y = y) = \int_{-\infty}^{+\infty} x f_{X\mid Y}(x\mid y)dx$

Central limit theorem

$$Z_n = \frac{S_n - n\mu}{\sqrt{n\sigma^2}} = \frac{\bar{X}_n - \mu}{\sqrt{\sigma^2/n}} \xrightarrow{\mathcal{D}} \mathrm{N}(0,1)$$

Sample mean	$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$
Sample variance	$s_n^2 = s_X^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2$
Two-sided C.I. for μ with known σ^2	$\left(\bar{X}_n - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\ ,\ \bar{X}_n + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$
Two-sided C.I. for μ with unknown σ^2	$\left(\bar{X}_n - t_{\alpha/2,n-1}\frac{s_n}{\sqrt{n}}\ ,\ \bar{X}_n + t_{\alpha/2,n-1}\frac{s_n}{\sqrt{n}}\right)$
Two-sided C.I. for σ^2	$\left(\frac{(n-1)s_n^2}{\chi^2_{\alpha/2,n-1}}\ ,\ \frac{(n-1)s_n^2}{\chi^2_{1-\alpha/2,n-1}}\right)$