Written Final Exam

Given name: ______________________ **Family name:** ______________________

User ID: ______________________ **Signature:** ______________________

Instructions

You have 90 minutes to complete the exam. You are **not allowed to cooperate or consult with anyone** while working on the exam. Remember that copying is easy to detect and that CTU in Prague considers cheating a very serious issue.

You must **show all your work and justify your methods** to obtain full score. Simplify your answers. You do not need to evaluate expressions such as e^3, $\ln 5$, $\sqrt{3}$, 2π. Do not use scratch paper. Use the back of the previous page if additional space is needed. **Calculators, organizers or computers are not allowed.** During the exam all cell phones and pagers brought into the classroom must be in your backpack, put in "silent" mode. The exam is worth a total of 60 points. A minimum of 30 points is needed to pass.

Achieved points:

Question	Max	Yours
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total:		

1. In a seven-floor building three people get in elevator in the first floor. Each of them can get off the elevator with the same probability in an arbitrary floor (beginning from the second floor). Find the probability that all passengers get off the elevator at the same floor.

2. Suppose we roll two balanced six-sided dice. Let X be a random variable denoting the smaller of both rolled numbers, i.e., $X = \min\{\text{roll } 1, \text{roll } 2\}$.

   (a) Find the distribution function of X and sketch its graph.

   (b) Find the expectation of X.

   (c) Find $\mathrm{P}(X = 2 | X \leq 2)$.

3. Let $X \sim \mathrm{Exp}(2)$ and $Y \sim \mathrm{Exp}(3)$ be *independent* random variables with exponential distributions with parameters $\lambda_X = 2$ and $\lambda_Y = 3$.

   (a) Find the joint density of the random vector (X, Y).

   (b) Find the probability $\mathrm{P}(X > 0 \cap Y < 1)$.

   (c) Are random variables X and Y non-correlated? Explain thoroughly.

   (d) Find $\mathrm{E}(X|Y = 1)$.

4. We are rolling a balanced six-sided die (probability of each outcome is $1/6$) and we record the relative frequency (number of successes over number of attempts) of number one.

   (a) Find the expected value and variance of the relative frequency of number one in n rolls.

   (b) How many rolls do we have to make, so that the relative frequency of the number one is below $1/4$ with the probability of $95\ \%$? *Use the Central Limit Theorem!*

5. Let $X_1, \ldots, X_n$ be a random sample from a uniform distribution on the interval $(1, 1+\theta)$, where $\theta > 0$ is a distribution parameter. By using the method of moments find an estimate for the parameter θ.

6. Suppose we have a random sample of 16 observed values from a normal distribution. The sample mean and the sample standard deviation are respectively $\bar{X}_n = 10.3$ and $s_n = 1.2$.

   In the answer bellow you don't need to exactly compute numerically the bounds of intervals, but only to *insert the correct values in the correct formulas*.

   (a) Find the two-sided confidence interval for the expected value with 99% confidence.

   (b) Find the two-sided confidence interval for the variance σ^2 with 99% confidence.