BIE-PST	**Written Final Exam**	Winter 2023/24
Probability and Statistics	FIT CTU in Prague	Jan 25, 2024

Given name: ____________________ **Family name:** ____________________

User ID: ____________________ **Signature:** ____________________

Instructions

You have 90 minutes to complete the exam. You are **not allowed to cooperate or consult with anyone** while working on the exam. Remember that copying is easy to detect and that CTU in Prague considers cheating a very serious issue.

You must **show all your work and justify your methods** to obtain full score. Simplify your answers. You do not need to evaluate expressions such as e^3, $\ln 5$, $\sqrt{3}$, 2π. Do not use scratch paper. Use the back of the previous page if additional space is needed. **Calculators, organizers or computers are not allowed.** During the exam all cell phones and pagers brought into the classroom must be in your backpack, put in “silent” mode. The exam is worth a total of 60 points. A minimum of 30 points is needed to pass.

Achieved points:

Question	Max	Yours
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total:		

1. In our faculty, statistically among every five students there are 4 male students and one female student. 75 % of girls have long hair, and among every 20 boys 3 have long hair too.

   (a) Find the overall percentage of long-haired students.

   (b) Suppose we sit in a lecture room and a person with long hair is sitting in front of us. What is the probability that this person is a girl?

2. Let X be a continuous random variable with the density

$$f(x) = \begin{cases} c\,x & x \in (0,2) \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Compute the constant c, so that $f(x)$ is truly a density.

(b) Find the expectation of X.

(c) Find the distribution function of X and sketch its graph.

(d) Find the median of X, i.e., such number m, for which $\mathrm{P}(X \leq m) = \frac{1}{2}$.

3. Suppose we observe a random variable X which follows the Normal distribution $\mathrm{N}(\mu, \sigma^2)$ with the parameters $\mu \in \mathbb{R}$ and $\sigma^2 > 0$. We know that $P(X \leq 5) = 0.5$ and $P(X \leq 8) = 0.8$. Find the values of the parameters μ and σ^2.

4. Suppose we perform 400 independent rolls with an unbalanced die. The die is unbalanced so that the probability of rolling a six is $1/5$.

   (a) Find the expectation and the variance of the **number of sixes** out of this 400 rolls.

   (b) What is the probability that out of the 400 rolls we have rolled at least 90 sixes? *Use the Central Limit Theorem!*

5. Let $X_1, \ldots, X_n$ be a random sample from a distribution with an unknown parameter θ.

   (a) Describe the steps of the method of moments in general and explain, why does this method produce reasonable estimates.

   (b) Find the estimate of the parameter $\theta \in (3, \infty)$ of the continuous uniform distribution on the interval $(3, \theta)$ using the method of moments.

6. We are analyzing how many email messages are needed to get a helpdesk ticket solved. We asked nine customers. Six customers needed just one email, two customers needed three emails and one needed six emails.

   (a) Find point estimates for the expectation and the variance of the number of needed emails.

   (b) The helpdesk department says, that on average, at most 1.5 emails are needed for solving one ticket. Design a statistical test of this claim and perform it on level of significance 5 %. Explain your steps in detail, along with the selection of hypotheses. For computations use a suitable approximation or rounding.