Given name: ______________________ **Family name:** ______________________

User ID: ______________________ **Signature:** ______________________

Instructions

You have 90 minutes to complete the exam. You are **not allowed to cooperate or consult with anyone** while working on the exam. Remember that copying is easy to detect and that CTU in Prague considers cheating a very serious issue.

You must **show all your work and justify your methods** to obtain full score. Simplify your answers. You do not need to evaluate expressions such as e^3, $\ln 5$, $\sqrt{3}$, 2π. Do not use scratch paper. Use the back of the previous page if additional space is needed. **Calculators, organizers or computers are not allowed.** During the exam all cell phones and pagers brought into the classroom must be in your backpack, put in "silent" mode. The exam is worth a total of 60 points. A minimum of 30 points is needed to pass.

Achieved points:

Question	Max	Yours
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total:		

1. A sports club registered 20 runners for the qualification for running the olympic marathon: 4 of them are excellent, 6 are very good and 10 are good. The probability that a runner accomplishes the limit for the olympic games is 0.9 for an excellent runner, 0.5 for a very good runner and 0.2 for a good runner. What is the probability that a randomly chosen runner will accomplish the limit?

2. Let X be a continuous random variable with density

$$f(x) = \begin{cases} c \cdot x \cdot (x-1) & \text{for } x \in [1,2], \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the constant c so that f truly is a density.

(b) State the definition and find the expected value $\mathrm{E}(X)$.

(c) Find the probability $\mathrm{P}(X < \mathrm{E}(X))$.

3. Let X be a random variable with the normal distribution with expectation $\mu = 100$ and variance $\sigma^2 = 400$. Let $Y = 4X - 300$.

   (a) Find the probability $\mathrm{P}(Y > 140)$.

   (b) Find the covariance and correlation coefficient between X and Y.

4. The capacity of a boat is 40 passengers. Suppose that the weight of people is random with expectation of $\mu = 80$ kg and standard deviation $\sigma = 20$ kg.
   (a) Find the expectation and variance of the total weight of all passengers if there are all 40 of them on board.

   (b) What must be the weight capacity of the boat, so that it will not sink with a probability of $99\ \%$ if there are all 40 passengers on board? *Use the Central Limit Theorem!*

5. Let $X_1, \ldots, , X_n$ be a random sample from a distribution with unknown parameters.

   (a) Describe the steps of the method of moments in general and explain, why does this method produce reasonable estimates.

   (b) Consider a random variable X with the expectation and variance of

$$\operatorname{E} X = \alpha + \beta \qquad \text{and} \qquad \operatorname{var} X = \beta^2,$$

   where $\alpha \in \mathbb{R}$ and $\beta > 0$ are unknown parameters. Estimate α and β using the method of moments.

6. Suppose we have a random sample of size 25 observed values from a normal distribution with known variance $\sigma^2 = 16$. We have computed the sample mean as $\bar{X}_n = 10$.

   (a) Find the two-sided 90% confidence interval for the expectation μ.

   (b) How large should be the random sample, so that the two-sided 95% confidence interval for μ would have the same width as the interval computed above?