BIE-PST **Written Final Exam** Winter 2023/24

Probability and Statistics FIT CTU in Prague Feb 02, 2024

Given name: ______________________ **Family name:** ______________________

User ID: ______________________ **Signature:** ______________________

Instructions

You have 90 minutes to complete the exam. You are **not allowed to cooperate or consult with anyone** while working on the exam. Remember that copying is easy to detect and that CTU in Prague considers cheating a very serious issue.

You must **show all your work and justify your methods** to obtain full score. Simplify your answers. You do not need to evaluate expressions such as e^3, $\ln 5$, $\sqrt{3}$, 2π. Do not use scratch paper. Use the back of the previous page if additional space is needed. **Calculators, organizers or computers are not allowed.** During the exam all cell phones and pagers brought into the classroom must be in your backpack, put in “silent” mode. The exam is worth a total of 60 points. A minimum of 30 points is needed to pass.

Achieved points:

Question	Max	Yours
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total:		

1. From previous experience we know that there is 40 per cent probability of acceptance of a paper to a given journal. After some time the reviewer ask for rewriting of some parts of your work. What is your probability of accepting now, if you know that in the past the editor asked for rewriting 70 per cent of the papers which were accepted and 40 per cent of the papers which were rejected?

2. Let X be a continuous random variable with the density

$$f(x) = \begin{cases} c \cdot (1+x) & \text{for } x \in [0,2], \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the constant c, so that f truly is a density.

(b) Find the expected value $\mathrm{E}(X)$.

(c) Find the probability $\mathrm{P}(X < \mathrm{E}(X))$.

3. Suppose that X and Y have a joint discrete distribution, with probabilities of possible combinations given in the following table:

$P(X = x \cap Y = y)$		Y		
		-1	0	1
	0	$1/6$	0	$1/6$
X	1	0	$1/6$	$1/6$
	2	0	$1/3$	0

(a) Find the marginal distribution of X and Y.

(b) Find the covariance of X and Y.

(c) Decide and explain whether X and Y are non-correlated. Decide and explain whether X and Y are independent.

4. A random number generator generates independent numbers from the continuous uniform distribution on the interval $[0, 1]$. So far it has generated 300 values.

   (a) Find the expectation and variance of the sum of the generated numbers.

   (b) What is the probability that the sum of the generated numbers will be larger than 160? *Use the Central Limit Theorem!*

5. Let $X_1, \dots, X_n$ be a random sample from a distribution with an unknown parameter θ.

   (a) Explain the maximum likelihood method of parameter estimations, step by step.

   (b) Find the estimate of the expectation θ of the normal distribution $\mathrm{N}(\theta, 1)$ by using the maximum likelihood method.
   Hint: The density of the normal distribution $\mathrm{N}(\mu, \sigma^2)$ *is* $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ *for all* $x \in \mathbb{R}$.

6. Consider a random sample of 9 values from a normal distribution. Using the measured values we computed the 90% two-sided confidence interval for the expectation as $(-2.72, 4.72)$.

   (a) Using this information compute the point estimates of the expectation and variance of the distribution.

   (b) Perform a test of the hypothesis $H_0 : \mu = 5$ against the alternative $H_A : \mu < 5$ on the level of significance 10 %. Explain in detail.