NIE-MPI, Mathematics for Informatics - Homework no. 1

Instructions:

- You should try to solve all the exercises. Even if you do not do all the exercises, you can get all the points.
- Presentation is taken into account; correct results themselves are not enough. The reasoning on how the result was found should be clearly visible.
- Comment your calculations in a reasonable way: the reader should understand what you do and *why*. The solution should be "possible to read", not "needed to decrypt".
- Do not answer unasked questions. It is important to know what is needed to solve the problem and what is not needed.
- If you use a result from another source than the lectures and tutorials, cite your source properly (do not forget to cite used software if applicable).
- The homework should be sent by email at `dolcefra@fit.cvut.cz` before Monday October 28th, 2024.

Exercise 1. Find all minima, maxima and saddle points of the following functions:

(a) $f(x, y) = 4x^2 + y^2 - xy$,

(b) $f(x, y) = \frac{1}{3x} - 5xy$,

(c) $f(x, y) = (x - y - 1)^2$,

(d) $f(x, y) = x^2 \ln y - y^2 \ln y - \frac{y^2}{2} + \sqrt{e}$,

(e) $f(x, y, z) = x^3 + y^4 - 9 \ln x - 16 \ln y$.

Exercise 2. Find minima, maxima and saddle points of the following functions:

(a) $f(x, y) = x^2 - 2y^2 + 6xy + 4$ subject to $x + y = 2$,

(b) $f(x, y, z) = xz + yz + z$ subject to $xyz = 1$.

Exercise 3. Calculate

$$\iint_D (x^3 y + x^2 y^2 - 1)\,\mathrm{d}x\,\mathrm{d}y$$

where D is equal to:

(a) $[0, 2] \times [0, 1]$;

(b) the triangle with vertices $(0,1),(1,1)$ and $(0,3)$;

(c) the bounded subset of $(\mathbb{R}^+)^2$ which is delimited by the x-axis, the curve having equation $y = 2x - x^2$ and the line having equation $y = 4x - 4$.