BIE-PST Cheat Sheet

FIT – CTU in Prague
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Probability	$\mathrm{P}(A) = \frac{\lvert A\rvert}{\lvert\Omega\rvert} = \frac{\text{\# of favorable outcomes}}{\text{\# of all outcomes}}$ or $\frac{\text{size of favorable outcomes}}{\text{size of all outcomes}}$
Conditional probability	$\mathrm{P}(A\vert B) = \dfrac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}$
Multiplicative law	$\mathrm{P}(A_1\cap\cdots\cap A_n) = \mathrm{P}(A_1)\,\mathrm{P}(A_2\vert A_1)\cdots\mathrm{P}(A_n\vert A_1\cap\cdots\cap A_{n-1})$
Law of total probability	$\mathrm{P}(A) = \sum_j \mathrm{P}(A\vert B_j)\mathrm{P}(B_j)$
Bayes' Theorem	$\mathrm{P}(B_i\vert A) = \dfrac{\mathrm{P}(A\vert B_i)\,\mathrm{P}(B_i)}{\sum_j \mathrm{P}(A\vert B_j)\mathrm{P}(B_j)}$
Independence	$\mathrm{P}(A\cap B) = \mathrm{P}(A)\,\mathrm{P}(B)$

Distribution function	$F_X(x) = \mathrm{P}(X\le x)$	
	Discrete random variable X	Continuous random variable X
Probabilities of values / Density:	$\mathrm{P}(X=x)$	$f_X(x)$
Expectation: $\mathrm{E}\,X$	$\sum_k x_k\,\mathrm{P}(X=x_k)$	$\int_{-\infty}^{+\infty} x f_X(x)\,\mathrm{d}x$
Expectation of a function: $\mathrm{E}\,g(X)$	$\sum_k g(x_k)\,\mathrm{P}(X=x_k)$	$\int_{-\infty}^{+\infty} g(x) f_X(x)\,\mathrm{d}x$
Variance X: $\mathrm{var}(X)$	$\mathrm{E}(X-\mathrm{E}\,X)^2 = \mathrm{E}\,X^2 - (\mathrm{E}\,X)^2$	

Distribution *support*	probability / density $\mathrm{P}(X=k)$ / $f_X(x)$	distribution $F_X(x)=\mathrm{P}(X\le x)$	expectation $\mathrm{E}\,X$	variance $\mathrm{var}\,X$
Bernoulli $k=0,1$	$\begin{cases} p, & k=1 \\ 1-p, & k=0 \end{cases}$	$0;\ 1-p;\ 1$	p	$p(1-p)$
Binomial $k=0,\dots,n$	$\binom{n}{k}p^k(1-p)^{n-k}$	$\sum_{i=0}^{\lfloor x\rfloor}\binom{n}{i}p^i(1-p)^{n-i}$	np	$np(1-p)$
Geometric $k=1,2,\dots$	$(1-p)^{k-1}p$	$1-(1-p)^{\lfloor x\rfloor}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Poisson $k=0,1,\dots$	$\frac{\lambda^k}{k!}e^{-\lambda}$	$e^{-\lambda}\sum_{i=0}^{\lfloor x\rfloor}\frac{\lambda^{\lfloor x\rfloor}}{i!}$	λ	λ
Uniform $a\le x\le b$	$\frac{1}{b-a}$	$\frac{x-a}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential $x\ge 0$	$\lambda e^{-\lambda x}$	$1-e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Normal $x\in\mathbb{R}$	$\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$\Phi(x)$	μ	σ^2

Joint distribution function $F_{X,Y}(x,y) = \mathrm{P}(X \le x \cap Y \le y)$	
Discrete X and Y	*Continuous X and Y*
Joint probabilities $\mathrm{P}(X = x \cap Y = y)$	Joint density $f_{X,Y}(x,y)$
Marginal distribution X	
$\mathrm{P}(X = x) = \sum_y \mathrm{P}(X = x \cap Y = y)$	$f_X(x) = \int_{-\infty}^{+\infty} f_{X,Y}(x,y)dy$
Independence of X and Y	
$\mathrm{P}(X = x \cap Y = y) = \mathrm{P}(X = x)\,\mathrm{P}(Y = y)$	$f_{X,Y}(x,y) = f_X(x) f_Y(y)$
Expectation of $\mathrm{E}\, g(X,Y)$	
$\sum_{x,y} g(x,y)\,\mathrm{P}(X = x \cap Y = y)$	$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(x,y) f_{X,Y}(x,y)dxdy$
Conditional distribution of X given $Y = y$:	
$\mathrm{P}(X = x \| Y = y) = \frac{\mathrm{P}(X = x \cap Y = y)}{\mathrm{P}(Y = y)}$	$f_{X\|Y}(x\|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$
Conditional expectation of X given $Y = y$:	
$\mathrm{E}(X\|Y = y) = \sum_x x\,\mathrm{P}(X = x\|Y = y)$	$\mathrm{E}(X\|Y = y) = \int_{-\infty}^{+\infty} x f_{X\|Y}(x\|y)dx$

Central Limit Theorem

$$Z_n = \frac{S_n - n\mu}{\sqrt{n\sigma^2}} = \frac{\bar{X}_n - \mu}{\sqrt{\sigma^2/n}} \xrightarrow{\mathcal{D}} \mathrm{N}(0,1)$$

Sample mean	$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$
Sample variance	$s_n^2 = s_X^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2$
Two-sided C.I. for μ with known σ^2	$\left(\bar{X}_n - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\ ,\ \bar{X}_n + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right)$
Two-sided C.I. for μ with unknown σ^2	$\left(\bar{X}_n - t_{\frac{\alpha}{2},n-1}\frac{s_n}{\sqrt{n}}\ ,\ \bar{X}_n + t_{\frac{\alpha}{2},n-1}\frac{s_n}{\sqrt{n}}\right)$
Two-sided C.I. for σ^2	$\left(\frac{(n-1)s_n^2}{\chi^2_{\frac{\alpha}{2},n-1}}\ ,\ \frac{(n-1)s_n^2}{\chi^2_{1-\frac{\alpha}{2},n-1}}\right)$