

NIE-MPI: Tutorial 3

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3.1 Homomorphisms and isomorphisms

Exercise 3.1. Which of the following mappings is a homomorphism and which is an isomorphism from the given group to the given group?

- (a) $f(n) = 3n + 2$, from $(\mathbb{Z}, +)$ to $(\mathbb{R}, +)$;
- (b) $f(x) = 2^x$, from $(\mathbb{R}, +)$ to $(\mathbb{R}^+, \cdot)$;
- (c) $f(A) = A_{1,1}$, from the group of $n \times n$ matrices with the matrix addition (element-wise), denoted $(M, +)$, to $(\mathbb{R}, +)$;
- (d) $f(A) = A_{1,1}$, from the group of $n \times n$ regular matrices with the matrix multiplication, denoted $(M_{\text{reg}}, \cdot)$, to $(\mathbb{R} \setminus \{0\}, \cdot)$.

Exercise 3.2. Find some homomorphism from $(M_{\text{reg}}, \cdot)$ to $(\mathbb{R} \setminus \{0\}, \cdot)$.

Exercise 3.3. Is $\mathbb{Z}_7^\times$ isomorphic with $\mathbb{Z}_6^+$? If yes, find an isomorphism.

Exercise 3.4. How can we find an isomorphism between the groups $\mathbb{Z}_p^\times$ and $\mathbb{Z}_{p-1}^+$ when p is a prime number? How many different isomorphisms exist?

3.2 Permutations

Exercise 3.5. Let us consider the two following permutations in S_5 :

$$f = (2\ 4\ 5\ 1\ 3) \quad \text{and} \quad g = (5\ 4\ 3\ 2\ 1).$$

- (a) Find $g \circ f$.
- (b) What is the order of the subgroup $\langle f \rangle$ of S_5 ? And the order of $\langle g \rangle$?
- (c) Find $f^{37} \circ g^{42}$.