

NIE-MPI: Tutorial 5

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5.1 Functions

Exercise 5.1. Let $f(x) = \sin(x)$ and $g(x) = (x - 3)^3$. Find a formula for the following composite functions:

- (a) $(f \circ g)(x)$,
- (b) $(g \circ f)(x)$,
- (c) $(f \circ g^{-1})(x)$,
- (d) $(g^{-1} \circ f)(x)$.

5.2 Derivatives

Exercise 5.2. Find the derivative of the following functions:

- (a) $(x^4 + 3x^3)x^8$,
- (b) e^{2x} ,
- (c) $\frac{x+3}{x^2}$,
- (d) $\ln((x+4)^{15})$,
- (e) $\sin^2 x + \cos^2 x$,
- (f) xe^{2x} ,
- (g) e^{x^2} ,
- (h) x^x .

Exercise 5.3. Let P be the set of all real polynomials. Is the set P closed under differentiation? In other words: is it true that $p \in P \Rightarrow p' \in P$?

Exercise 5.4. Let $p(x) = \sum_{k=0}^n a_k x^k$ be a polynomial of degree n (i.e., $a_n \neq 0$), where $n \in \mathbb{N}$. Find the n -th derivative $p^{(n)}$.

Exercise 5.5. Find the n -th derivative of $\sin x$. Try to express the result as simply as possible.

5.3 Partial derivative

Exercise 5.6. Find the following partial derivatives:

- (a) $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for

$$f(x, y) = xy + e^x \cos y,$$

- (b) $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for

$$f(x, y) = x^2 y^3 + x^3 y^4 - e^{xy^2},$$

- (c) find the value of $\frac{\partial f}{\partial z}$ at point $(1, 2, 3)$ for

$$f(x, y, z) = \sin\left(\frac{xy}{z}\right),$$

- (d) $\frac{\partial f}{\partial x}$ for

$$f(x, y) = e^{-x^2 - y^2},$$

- (e) $\frac{\partial f}{\partial x}$ for

$$f(x, y) = \ln(x^2 + y^2 + 1),$$

- (f) $\frac{\partial f}{\partial x}$ for

$$f(x, y) = \frac{1}{x^3 + y^3}.$$

Exercise 5.7. Find $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$ for:

(a) $f(x, y) = x^2 y^2,$

(b) $f(x, y) = \sin(xy),$

(c) $f(x, y) = xy^2 - ye^{-x} - \cos(x - y).$

Exercise 5.8. Find the mixed partial derivatives $\frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y \partial x}$ for:

(a) $f(x, y, z) = e^{xz} + y \cos x,$

(b) $f(x, y, z) = z \cos(xy) + x \sin(yz).$