

NIE-MPI: Tutorial 6

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6.1 Critical points

Exercise 6.1. Determine all the points where the function $\sqrt{x^2 + y^2}$ does not have partial derivatives.

Exercise 6.2. Find all the critical points of the following functions.

- (a) $f(x, y) = x^2 + 2y^2$,
- (b) $f(x, y, z) = 3x^2 + xy + 3zy$,
- (c) $f(x, y) = e^{-x^2 - 7y^2 + 3}$,
- (d) $f(x, y) = x^2 + y^2 + 3xy + 10$,
- (e) $f(x, y) = \sin(x^2 + y^2)$,
- (f) $f(x, y) = x^2 + xy^3 - \ln(x)$.

6.2 Hessian matrix, extremal values

Exercise 6.3. Find the Hessian matrix of the following functions.

- (a) $f(x, y) = x^2y^2$,
- (b) $f(x, y) = e^{-(x+y)}$,
- (c) $f(x, y, z) = x^2 + y^3 + z^4$.

Exercise 6.4. For which values of a is the matrix

$$\begin{pmatrix} 4 & 1 \\ 1 & a \end{pmatrix}$$

positively (semi)definite, negatively (semi)definite, indefinite.

6.3 Minima and maxima

Exercise 6.5. Find all local minima and maxima and saddle points of the following functions.

(a) $f(x, y) = x^2 + 3xy + y^2 + 16,$

(b) $f(x, y) = 3x^2 - 5xy + 3y^2,$

(c) $f(x, y) = (x^2 - y^2)e^{(-x^2 - y^2)/2},$

(d) $f(x, y) = e^{1+x^2-y^2},$

(e) $f(x_1, x_2, x_3) = x_1^2 + 3x_2^2 - 3x_1x_2 + 4x_2x_3 + 6x_3^2,$

(f) $f(x_1, x_2, x_3) = x_1x_3 + x_1^2 - x_2 + x_2x_3 + x_2^2 + 3x_3^2.$