NIE-MPI: Tutorial 7

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7.1 Constrained optimization

Exercise 7.1. Find all the local maxima and minima of the function $f(x, y) = 3x - 4y + 3$ subject to

$$x^2 + y^2 = 4.$$

Exercise 7.2. Find all the local maxima and minima of the function $f(x, y) = xy$ subject to

$$x + y = 1.$$

Exercise 7.3. Find all the local maxima and minima of the function $f(x, y) = x^2 + y^2$ subject to

$$\frac{x}{a} + \frac{y}{b} = 1,$$

where a and b are non-zero real numbers.

Exercise 7.4. Find all the local maxima and minima of the function $f(x, y) = 2x^2 - 2y^2$ subject to

$$y + e^{-x^2} = 1.$$

7.2 2-variate function integration

Exercise 7.5. Let $f(x, y) = e^{2x+y}$ and $D = [0, 1] \times [0, 3]$. Evaluate

$$\iint_D f(x, y)\, \mathrm{d}x\, \mathrm{d}y.$$

Exercise 7.6. Let $f(x, y) = \sin(x + y)$ and $D = [0, \pi] \times [0, 2\pi]$. Evaluate

$$\iint_D f(x, y)\, \mathrm{d}x\, \mathrm{d}y.$$

Exercise 7.7. Find the volume of the solid object delimited by the graph of

$$f(x, y) = x^2 + y^2$$

and by the planes $x = 0, x = 3, y = -1$ and $y = 1$ (the volume is positive for $z > 0$ and negative for $z < 0$).

Exercise 7.8. Let $f(x,y,z) = x + 2y + 3z$ and $D = [0,1] \times [-\frac{1}{2}, 0] \times [0, \frac{1}{3}]$. Evaluate

$$\iiint_D f(x,y,z)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z.$$

Exercise 7.9. Let $f(x,y,z) = e^{x+y+z}$ and $D = [0,1] \times [0,1] \times [0,1]$. Evaluate

$$\iiint_D f(x,y,z)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z.$$

Integrals over non-rectangular domain

Exercise 7.10. Evaluate

$$\iint_D (x+y)\,\mathrm{d}x\,\mathrm{d}y\,,$$

where D is the domain delimited by the graph of $y = x^2$ for $x \in [0, \frac{1}{2}]$ and the x-axis.

Exercise 7.11. Evaluate

$$\iint_D (x+y)^2\,\mathrm{d}x\,\mathrm{d}y$$

where D is the triangular surface with vertices $(0,0), (0,1)$ and $(2,2)$.

Exercise 7.12. Evaluate

$$\int_0^1 \int_x^1 xy\,\mathrm{d}y\,\mathrm{d}x\,.$$

Exercise 7.13. Evaluate

$$\int_0^1 \int_{1-y}^1 (x+y^2)\,\mathrm{d}x\,\mathrm{d}y\,.$$

Exercise 7.14. Evaluate

$$\iint_D (x-y)\,\mathrm{d}x\,\mathrm{d}y\,,$$

where D is the triangular surface with vertices $(0,0), (1,0)$ and $(2,1)$.