NIE-MPI: Tutorial 9

last update: August 25, 2025, 13:45 by Francesco Dolce

9.1 Numerical mathematics

Machine numbers

If not stated otherwise, we consider **single precision** and mathematical rounding.

Exercise 9.1. Are the following numbers machine numbers?

a) 10^{113}

b) $1 + 2^{-32}$

c) $\frac{1}{5}$

d) $\frac{3}{10}$

e) $\frac{3}{256}$

f) $2^{-20} - 16$

Exercise 9.2. Which number correspond, in base 10, to the following machine numbers? (we use the notation **(sign, mantissa, exponent)**)

a) (1, 10100000000000000000000, 01000010);

b) (0, 10100000000000000000000, 00000001);

c) (1, 10100000000000000000000, 00000000).

Exercise 9.3. Which are the closest neighbours, between the normalized machine numbers, of the normalized machine number 2^t?

Exercise 9.4. Consider a decimal machine in which 2 decimal digits are allocated to the significand (and we do not care about the exponent) and the approximation is done by rounding. Sum the following numbers $0,25$, $0,0034$, $0,00051$ a $0,061$ in the following order:

a) from the least to the greatest,

b) from the greatest to the least.

Compare to the exact result.

Exercise 9.5. Find the absolute and the relative errors of the following pair (a, α), where α is an approximate value of a:

- $(0.100, 0.110)$,
- $(0.100, 0.101)$,
- $(0, 500, 0.510)$,
- $(0.500, 0.501)$.

Exercise 9.6. Let x and y be normalized machine numbers. Which following statements are true if we suppose that no underflow or overflow happens (and we stay within normalized numbers)?

1. $\mathrm{fl}(x + y) = \mathrm{fl}(x) + \mathrm{fl}(y)$;
2. $\mathrm{fl}(x + y) = \mathrm{fl}(y + x)$;
3. $\mathrm{fl}((x + y) + z) = \mathrm{fl}(x + (y + z))$;

If a statement is not true, find a counterexample. If it is true, give an argument. If your answer depends on something else, mention it.