BIE-PST: Tutorial 11 – Winter 2025/2026 (FIT CTU)

Review - Hypothesis testing

A **hypothesis** is a statement, which can either be true or not. We want to decide on it's validity based on available data. Denote

- H_0 the null hypothesis,
- H_A the alternative hypothesis.

A **statistical test** is a decision rule based on observed data, with two possible outcomes:

- **reject** H_0 in favor of H_A (*if our data stand against* H_0 *and for* H_A),
- **do not reject** H_0 (*based on our data* H_0 *cannot be rejected, the data do not contradict* H_0).

In the decision process we can make an **error**:

	Decision	
Reality	reject H_0	don't reject H_0
H_0 holds	type I error	OK
H_0 doesn't hold	OK	type II error

Type I error is more severe (we are falsely incriminating) $\rightarrow$ its probability must be controlled:

- We choose a **level of significance** of the test α as the maximum possible probability of making type I error (most often $\alpha = 0.05$ or $\alpha = 0.01$).
- Denote $\beta = \mathrm{P}(\text{type II error})$. Then $1 - \beta$ is called the *power* of the test.

p-value (p-value) = highest achieved level

- The result of the test often given by statistical software.
- Probability of getting a result which stands against H_0 even more than the data at hand, if H_0 is actually valid.
- Rule of thumb: p-value $\leq \alpha \;\Rightarrow\;$ reject H_0 in favor of H_A

Decision rule

- Reject H_0 if the tested value is not in the corresponding $(1-\alpha)\%$ confidence interval.
- Equivalently: Reject H_0 if the p-value of the test is lower than α.
- Equivalently: Reject H_0 if the *test statistic* lies in the corresponding α-*critical region*.

EXERCISES 11 - HYPOTHESIS TESTING

1. A safety sensor regularly scans a room. If there is no movement in the room, the sensor gives a signal $X = W$, where W is a random variable with expected value 0. If there is a movement, the sensor gives a signal $X = W + \theta$, where $\theta > 0$ is an unknown constant. We have 35 observations and we computed symmetric confidence intervals for $\mu = \mathrm{E}\, X$ as follows:

$$\begin{array}{ll} \text{Symmetric 90\% confidence interval:} & (0.405, 5.394) \\ \text{Symmetric 95\% confidence interval:} & (-0.072, 5.872) \end{array}$$

Test the hypothesis $H_0 : \mu = 0$ versus the alternative $H_A : \mu > 0$, so that the probability of type I error is at most 5%. Which interval do you use and why?

2. Before a storm, the variance of the measured wind speeds usually grows. If a storm is coming, the variance σ^2 exceeds 4.5. Suppose that the wind speed measurements are normally distributed. We have computed the confidence intervals for σ^2 based on 200 measurements as:

$$\begin{array}{ll} \text{Symmetric 99\% confidence interval:} & (2.64, 4.43) \\ \text{Symmetric 98\% confidence interval:} & (2.7, 4.31) \end{array}$$

Is there a storm coming? Test the hypothesis $H_0 : \sigma^2 \leq 4.5$ versus $H_A : \sigma^2 > 4.5$ so that the probability of type I error is at most 1%. Which interval have you used and why?

3. We have records from pizza sales from previous 9 days:

41, 38, 56, 38, 63, 59, 52, 49, 46.

a) Based on the data estimate how many pizzas we are likely to sell on the next day?

b) Suppose that data come from the normal distribution. Find symmetric 90% and 95% confidence intervals for the expected value.

c) Why is the 95% confidence interval wider that the 90% one?

d) Can we reject the hypothesis that the expected value is 45?

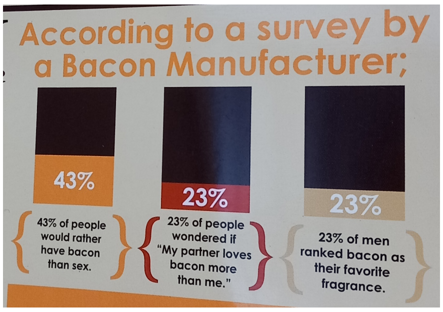
4. Suppose we observe a random sample of $n = 16$ values from the normal distribution. The sample mean and sum of second powers are

$$\bar{X}_{10} = 13 \qquad \text{and} \qquad \sum_{i=1}^{16} X_i^2 = 2708\,.$$

a) Test the hypothesis $H_0 : \mu = 15$ versus $H_A : \mu \neq 15$ on level of significance 5%.

b) Test the hypothesis $H_0 : \sigma^2 = 0.5$ versus $H_A : \sigma^2 \neq 0.5$ on level of significance 5%.

5. A Bacon Manufacturer conducted a survey asking people whether they considered bacon a source of jalousy in their relationship. Out of 97 people 14 answered positively. They then published the study stating that 23% of people considered bacon a potential romantic rival.



a) Can we reject their statement on level of significance 5%? And of level of significance 1%?

b) What if the stement was "more than 23%" (again for $\alpha = 5\%$ and $\alpha = 1\%$)?

Additional exercises - Hypothesis testing

One sample test of the expectation and variance

6. A car manufacturer states that the average fuel consumption of a new model is 6 liters per 100 kilometers. We have tested 20 cars and observed that the average consumption was $\bar{X}_n = 6.8$ liters per 100 km with a sample variance of $s_n^2 = 2.56$.

a) On level of significance $\alpha = 5\%$, perform a test to determine whether the manufacturer's statement is true or whether the actual consumption is significantly higher.

b) Repeat for $\alpha = 1\%$.

7. A production line makes steel parts. The variance of their lengths shouldn't be larger than 300 μm^2. Based on a sample of 15 parts we have computed the sample variance as $s_n^2 = 580$ μm^2.

a) On the level of significance $\alpha = 5\%$, perform a test to determine whether the production line is operating with sufficient precision, or whether it should be recalibrated because the variance is too large.

b) Repeat for $\alpha = 1\%$.

8. We are doing a research to determine what is the actual volume of beer in one glass served in one specific bar. We bought ten beer and the volume was (in liters):

$$0.510,\ 0.462,\ 0.491,\ 0.466,\ 0.451,\ 0.503,\ 0.475,\ 0.487, 0.512, 0.505.$$

Suppose that the volume of beer in one glass follows the normal distribution $\text{N}(\mu, \sigma^2)$ and the measurements are independent.

a) Estimate the expected volume μ of beer in one glass.

b) Estimate the probability of receiving an undersized beer (less than 0.5 l).

c) As customers, we would like to know whether the bartender isn't cheating on us. Perform a test of the hypothesis whether the real expected volume of beer truly is 0.5 liters, against the one-sided alternative that the real expected volume is less than 0.5 l. Use $\alpha = 5\%$.

d) Repeat for $\alpha = 1\%$.